

6.5: Factoring Cubic Polynomials

[Algebra 2(Y)]

HCPS III

- **Standard 10:** Patterns, Functions, and Algebra: SYMBOLIC REPRESENTATION: Use symbolic forms to represent, model, and analyze mathematical situations.
- **Benchmark MA.All.10.4:** Factor polynomials representing perfect squares, the difference in squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

Goal: Factor cubic polynomials and solve cubic equations.

Special Product Patterns

Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Example

$$\begin{aligned}x^3 + 8 &= x^3 + 2^3 \\ &= (x + 2)(x^2 - 2x + 4)\end{aligned}$$

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example

$$\begin{aligned}27x^3 - 1 &= 3x^3 - 1^3 \\ &= (3x + 1)(9x^2 + 3x + 1)\end{aligned}$$

Example 1: Factor the Sum or Difference of Two Cubes

Factor each expression.

a.) $27x^3 + 125$

b.) $x^3 + 64$

c.) $d^3 - 125$

d.) $8p^3 - q^3$

Example 2: Factor Polynomials

Factor each expression.

a.) $x^3 - 5x^2 + 6x$

b.) $16x^4 - 2x$

c.) $10x^4 - 10$

d.) $3m^{12} + 48m^7 - 51m^2$

Example 3: Factor By Grouping

Factor each expression.

a.) $x^2(x - 1) - 9(x - 1)$

b.) $x^3 - 2x^2 - 16x + 32$

c.) $4x^2(x - 3) - 25(x - 3)$

d.) $x^3 + 4x^2 - 36x - 144$

Example 4: Solve a Cubic Equation by Factoring

Solve each equation by factoring.

a.) $2x^3 - 14x^2 = -24x$

b.) $3x^3 - 6x = -3x^2$

c.) $x^3 - 6x^2 + 12 = 2x$

d.) $x^3 + 5x^2 - 3x = 15$