2.1: Functions and Their Graphs

ADP Content Standards:
- **P1.a:** Determine key characteristics of quadratic functions and their graphs.

CCSS for High School Mathematics:
- **F.IF.1:** Understand the concept of a function and use function notation
  Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).
- **F.IF.7:** Analyze functions using different representations
  Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Objective:
- To identify and graph functions.

Relations

A **relation** is a mapping, or pairing of input values with output values.

**Domain**
- Input values
- Independent variable
- Usually \( x \)-coordinates

**Range**
- Output values
- Dependent variable
- Usually \( y \) or \( f(x) \) - coordinates

**Ordered Pairs**
- \((x, y)\)
- \((-3, 4)\)
- \((3, -1)\)
- \((4, -1)\)
- \((4, 3)\)

**Mapping Diagram**

**Table of Values**

**Graph**

Arrows show how to pair each input with an output.
Example 1: Finding Domain and Range
What are the domain and range of each relation?

a.) \{ (5, 9), (4, 11), (3, 13), (2, 15) \}

b.) \{ (-3, 14), (0, 7), (2, 0), (9, -18) \}

Identifying Functions

A function is a relation in which each element of the domain corresponds with exactly one element of the range.

Example 2: Identifying Functions
Is the relation a function?

a.) Domain | Range
-3 | -2
0 | 1
4 | 7

b.) \{ (4, -1), (8, 6), (6, 6), (4, 1) \}
Vertical Line Test

Vertical Line Test for Functions

A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point.

Example 3: Using the Vertical-Line Test

Use the vertical-line test. Which graph(s) represent functions?

a.)

b.)

c.)
Graphing Functions

An *equation in two variables* can represent many functions.

**Graphing Equations in Two Variables**
1. Make a table of values, and write the ordered pairs.
2. Plot enough solutions to recognize a pattern.
3. Connect the points with a line or curve.

**Example 4: Graphing an Equation**
Graph $y = x + 1$ by using a table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$(x, y)$</th>
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![Graph of $y = x + 1$]