3.5: Solving Systems of Linear Equations in Three Variables

ADP Content Standards:

- ***E1.b:*** Express and solve systems of linear equations in three variables with and without the use of technology.
- ***E1.d:*** Recognize and solve problems that can be represented by single variable linear equations or inequalities or systems of linear equations or inequalities involving two or more variables. Interpret the solution(s) in terms of the context of the problem.

CCSS for High School Mathematics:

- ***A.REI.6: Solve systems of equations***
  Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Objective: To solve systems of linear equations in ***three variables*.**

Three-Dimensional Coordinate System

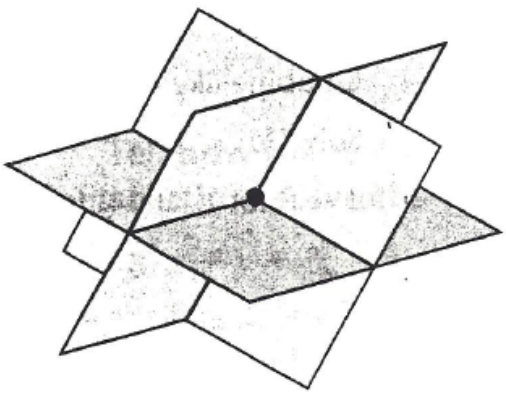
A ***linear equation in three variables*** contains variables *x*, *y*, and *z* in the form:

$$ax + by + cz = d$$

The ***solution*** to a system of linear in equations in three variables can be represented by an ***ordered triple*** (*x, y, z*).

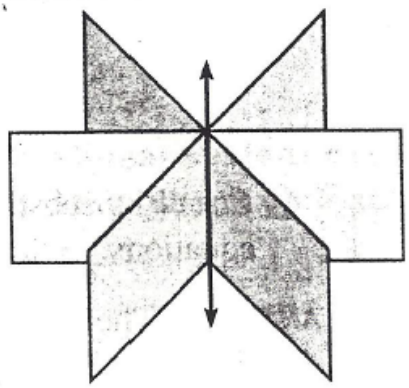
Exactly one solution

If the planes intersect in a single point, the system has exactly one solution.



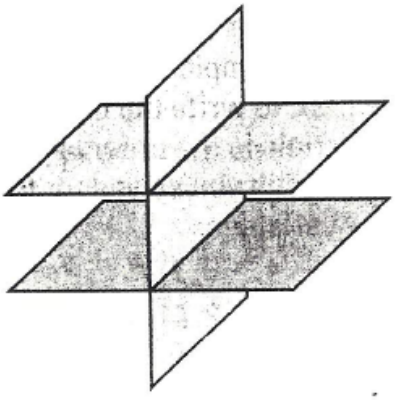
Infinitely many solutions

If the planes intersect in a line or a plane, the system has infinitely many solutions.



No solution

If the three planes do not intersect, the system has no solution.



Solving a System of Linear Equations in Three Variables: LINEAR COMBINATION METHOD (ELIMINATION)

1. Choose ***two equations*** and ***eliminate one variable***.
2. Choose ***another two equations*** and ***eliminate the same variable***.
3. Use the two ***new equations*** and ***perform a regular elimination***.
4. ***Substitute*** the values of the two solved variables into any of the original equations and ***find the last variable***'s value.

Example 1: Use the Linear Combination Method

Solve each system.

a.) $3x - 2y + 4z = 20$
$-x + 5y + 12z = 73$
$x + 3y - 2z = 1$

Solution (2, 3, 5)

b.) $2x + 5y - 3z = 9$
$4x - 3y + 3z = 7$
$x - 5y + 3z = 12$

Solution (7, -13, -20)

Solving a System of Linear Equations in Three Variables: SUBSTITUTION

1. Choose one equation and ***solve*** (isolate) for ***one variable***.
2. ***Substitute*** the isolated expression ***into another equation***.
3. ***Substitute*** the isolated expression ***into the remaining equation.***
4. Now that you have a ***system of two equations***, ***solve for the two variables*** using your preferred method to solve the system.
5. ***Substitute*** the values of the two solved variables into any of the original equations and ***find the last variable***'s value.

Example 2: Use the Substitution Method

Solution (2, -2, 1)

Solve each system.

a.) $x - y - z = 3$

$-x + 2y + 5z = -1$

$x + y + 4z = 4$

b.) $-x + z = 2$
$x + 2y - 3z = 2$
$3x + 4y - 2z = -3$

Solution (-3, 1, -1)