5.6: Geometry: Parallel and Perpendicular Lines [Algebra 1(X)]

Hawaii Content and Performance Standards (HCPS) III

- ***Standard 10***: Patterns, Functions, and Algebra: SYMBOLIC REPRESENTATION: Use symbolic forms to represent, model and analyze mathematical situations..
- ***Benchmark MA.AI.10.4:*** Determine the equation of a line when given the graph of the line, the slope and a point on the line, or two points on the line.

Common Core State Standards (CCSS) for High School Mathematics

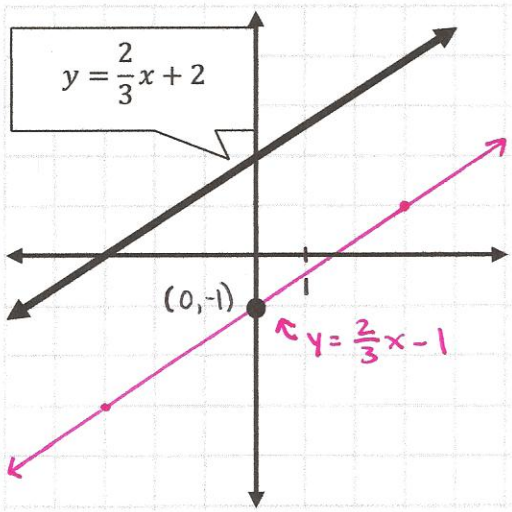
- ***A.CED.2: Create equations that describe numbers or relationships.***
  Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
- ***S.ID.7: Interpret linear models.***
  Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.*

Objectives(s):

- To write an equation of the line that passes through a given point, parallel to a given line.
- To write an equation of the line hat passes through a given point, perpendicular to a given line.

Parallel Lines (| |)

Two lines are ***parallel*** if they have same slopes.



In the graph to the left, **draw a line through the given point** that is **parallel** to the given line. $m = \frac{2}{3} = \frac{rise}{run}$

What is the **equation** of the line you drew in slope intercept form?

$m = \frac{2}{3}$ $b = -1$ * $y = mx + b$

$\boxed{y = \frac{2}{3}x - 1}$

Steps for Writing an Equation of a Parallel Line

1. **Determine** the **slope** (*m*) of the given line.
2. **Use** the **Point-Slope formula** and substitute *m* and the given point.
3. Write the new linear equation in **Slope-Intercept Form**.

Example 1: Parallel Line Through a Given Point

Write the slope-intercept form of an equation of the line that passes through the given point and is parallel to the graph of each equation. **REMINDER:** make sure your given line is in slope-intercept form.

a.) $(-1,-2); y = -3x - 2$

① Determine Slope

$m = -3$ ← from equation

② Point-Slope Form

$y - y_1 = m(x - x_1)$

$y + 2 = -3(x + 1)$

③ Slope-Intercept Form

$y + 2 = -3(x + 1)$

$y + \cancel{2} = -3x - 3$

$\underline{-\cancel{2}} \qquad \underline{-2}$

$\boxed{y = -3x - 5}$

b.) $(4,-2); y = \frac{1}{2}x - 7$

① $m = \frac{1}{2}$

② $y - y_1 = m(x - x_1)$

$y + 2 = \frac{1}{2}(x - 4)$

③ $y + 2 = \frac{1}{2}(x - 4)$

$y + \cancel{2} = \frac{1}{2}x - 2$

$\underline{-\cancel{2}} \qquad \underline{-2}$

$\boxed{y = \frac{1}{2}x - 4}$

c.) $(2,2); 3y = -2x + 6$

① $\frac{\cancel{3}y}{\cancel{3}} = -\frac{2x}{3} + \frac{6}{3}$

$y = -\frac{2}{3}x + 2 \qquad m = -\frac{2}{3}$

② $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{2}{3}(x - 2)$

③ $y - \cancel{2} = -\frac{2}{3}x + \frac{4}{3}$

$\underline{+\cancel{2}} \qquad \underline{+\frac{6}{3}}$

$\boxed{y = -\frac{2}{3}x + \frac{10}{3}}$

d.) $(2,2); 3x - 4y = -4$

① $3\cancel{x} - 4y = -4$

$\underline{-3\cancel{x}} \qquad \underline{-3x}$

$\frac{-\cancel{4}y}{-\cancel{4}} = \frac{-3x}{-4} - \frac{4}{-4}$

$y = \frac{3}{4}x + 1 \qquad m = \frac{3}{4}$

② $y - y_1 = m(x - x_1)$

$y - 2 = \frac{3}{4}(x - 2)$

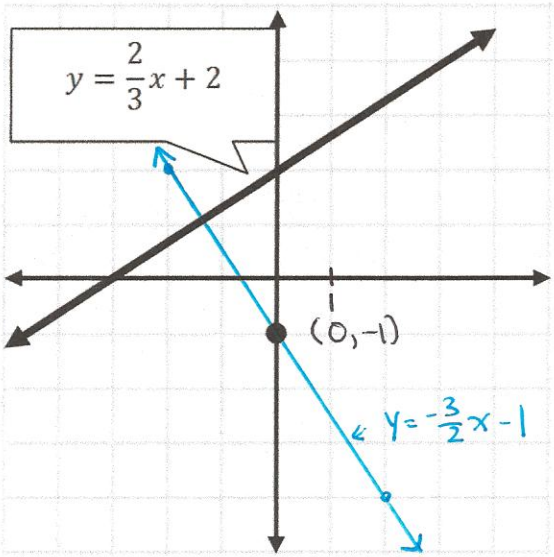
③ $y - \cancel{2} = \frac{3}{4}x - \frac{3}{2}$

$\underline{+\cancel{2}} \qquad \underline{+\frac{4}{2}}$

$\boxed{y = \frac{3}{4}x + \frac{1}{2}}$

Perpendicular Lines (⊥)

Two lines are perpendicular if their slopes are opposite reciprocals.



- In the graph to the left, the slope of the given line is $\frac{2}{3}$; it's perpendicular slope is $-\frac{3}{2}$.

$$+\frac{2}{3} \qquad -\frac{3}{2}$$

- In the graph to the left, **draw a line through the given point** that is **perpendicular** to the given line.
- What is the **equation** of the line you drew in slope-intercept form?

$m = -\frac{3}{2}$ $b = -1$ $\boxed{y = -\frac{3}{2}x - 1}$

Steps for Writing an Equation of a Perpendicular Line

1. **Determine** the **slope** (m) of the given line.
2. **Determine** the **perpendicular slope** ($m_\perp$) for the new line.
3. **Use** the **Point-Slope formula** and substitute $m_\perp$ and the given point.
4. Write the new linear equation in **Slope-Intercept Form**.

Example 2: Perpendicular Line Through Given Point

Write the slope-intercept form of an equation of the line that passes through the given point and is perpendicular to the graph of each equation.

a.) $(-3, -2);\ y = -\frac{1}{4}x + 3$

① Determine given slope

$m = -\frac{1}{4}$

② Determine Perpendicular Slope

$m_\perp = \frac{4}{1} = 4$

③ Point-Slope Formula

$y - y_1 = m(x - x_1)$

$y + 2 = 4(x + 3)$

④ Slope-Intercept Form

$y + 2 = 4x + 12$

$-2 \qquad -2$

$\boxed{y = 4x + 10}$

b.) $(-2, 0);\ y = x - 6$

① $m = 1$

② $m_\perp = -1$

③ $y - y_1 = m(x - x_1)$

$y - 0 = -1(x + 2)$

④ $\boxed{y = -x - 2}$

c.) $(-3, -2);\ x + 4y = 12$

① Determine Slope

$$\cancel{x} + 4y = 12$$
$$\underline{-\cancel{x}} \qquad \underline{-x}$$
$$\frac{\cancel{4}y}{\cancel{4}} = \frac{-x}{4} + \frac{12}{4}$$
$$y = -\frac{1}{4}x + 3$$
$$m = -\frac{1}{4}$$

② Determine Perpendicular Slope

$$m_{\perp} = \frac{4}{1} = 4$$

③ Point-Slope Formula

$$y - y_1 = m_{\perp}(x - x_1)$$
$$y + 2 = 4(x + 3)$$

④ Slope-Intercept Form

$$y + \cancel{2} = 4x + 12$$
$$\underline{-\cancel{2}} \qquad \underline{-2}$$
$$\boxed{y = 4x + 10}$$

d.) $(0, 6);\ 2y + 5x = 2$

① Determine Slope

$$2y + \cancel{5x} = 2$$
$$\underline{-\cancel{5x}} \qquad \underline{-5x}$$
$$\frac{\cancel{2}y}{\cancel{2}} = \frac{-5x}{2} + \frac{2}{2}$$
$$y = -\frac{5}{2}x + 1$$
$$m = -\frac{5}{2}$$

② Determine Perpendicular Slope

$$m_{\perp} = \frac{2}{5}$$

③ Point-Slope Formula

$$y - y_1 = m_{\perp}(x - x_1)$$
$$y - 6 = \frac{2}{5}(x - 0)$$

④ Slope-Intercept Form

$$y - \cancel{6} = \frac{2}{5}x$$
$$\underline{+\cancel{6}} \qquad \underline{+6}$$
$$\boxed{y = \frac{2}{5}x + 6}$$