4.2: Transformations on the Coordinate Plane

Hawaii Content & Performance Standards (HCPS) III:
- **Standard 8**: Geometry and Spatial Sense: TRANSFORMATIONS AND SYMMETRY: Use transformations and symmetry to analyze mathematical situations.

Common Core State Standards (CCSS) for High School Mathematics:
- **G.CO.5**: Experiment with transformations in the plane.
  Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Objectives:
- To transform figures by using reflections, translations, dilations, and rotations.
- To transform figures on a coordinate plane by using reflections, translations, dilations, and rotations.

Transform Figures

**Transformation**: the movement of a geometric figure.

**Preimage**: the position of the figure before the transformation.

**Image**: the position of the figure after the transformation.

There are four main types of transformations.
Types of Transformations

**Reflection**
A figure is flipped over a line.

![Reflection Diagram]

**Translation**
A figure slides.

![Translation Diagram]

**Dilation**
A figure is enlarged or reduced.

![Dilation Diagram]

**Rotation**
A figure is turned around a point.

![Rotation Diagram]

**Example 1: Identify Transformations**
Identify each as a *reflection, translation, dilation, or rotation*.

a.)

![a.) Diagram]

b.)

![b.) Diagram]

c.)

![c.) Diagram]

d.)

![d.) Diagram]
**Key Concept: Transforms Figures on the Coordinate Plane**

You can perform transformations on a coordinate plane by changing the coordinates of the points on a figure. The points on the translated figure are indicated by the prime symbol ‘ ′ to distinguish them from the original points.

<table>
<thead>
<tr>
<th>Name</th>
<th>Words</th>
<th>Symbols</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td>To reflect a point over the x-axis, multiply the y-coordinate by -1.</td>
<td>reflection over x-axis: (x, y) → (x, -y)</td>
<td><img src="image1" alt="Reflection over x-axis" /></td>
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<tr>
<td></td>
<td>To reflect a point over the y-axis, multiply the x-coordinate by -1.</td>
<td>reflection over y-axis: (x, y) → (-x, y)</td>
<td><img src="image2" alt="Reflection over y-axis" /></td>
</tr>
<tr>
<td>Translation</td>
<td>To translate a point by an ordered pair (a, b), add a to the x-coordinate and b to the y-coordinate.</td>
<td>(x, y) → (x + a, y + b)</td>
<td><img src="image3" alt="Translation" /></td>
</tr>
<tr>
<td>Dilation</td>
<td>To dilate a figure by a scale factor k, multiply both coordinates by k.</td>
<td>(x, y) → (kx, ky)</td>
<td><img src="image4" alt="Dilation" /></td>
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<td></td>
<td>If k &gt; 1, the figure is enlarged.</td>
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<tr>
<td></td>
<td>If 0 &lt; k &lt; 1, the figure is reduced.</td>
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<tr>
<td>Rotation</td>
<td>To rotate a figure 90° counter-clockwise about the origin, switch the coordinates of each point and then multiply the new first coordinate by -1.</td>
<td>90° rotation: (x, y) → (-y, x)</td>
<td><img src="image5" alt="90° Rotation" /></td>
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<tr>
<td></td>
<td>To rotate a figure 180° about the origin, multiply both coordinates of each point by -1.</td>
<td>180° rotation: (x, y) → (-x, -y)</td>
<td><img src="image6" alt="180° Rotation" /></td>
</tr>
</tbody>
</table>
**Example 2: Reflection**

A trapezoid with the vertices $W (-1, 4), X (4, 4), Y (4, 1),$ and $Z (-3, 1)$ is reflected over the $x$-axis.

a.) Find the coordinates of the vertices of the image.

b.) Graph trapezoid $WXYZ$ and its image $W'X'Y'Z'$.

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**Example 3: Translation**

A triangle with vertices $A (-2, 1), B (2, 4),$ and $C (1, 1)$ is translated 3 units to the right and 5 units down.

a.) Find the coordinates of the vertices of the image.

b.) Graph triangle $ABC$ and its image $A'B'C'$. 
**Example 4: Dilation**
A trapezoid with the vertices $E (-1, 2), F (2, 1), G (2, -1)$, and $H (-1, -2)$ is dilated by a scale factor of 2.

a.) Find the coordinates of the vertices of the image.

b.) Graph trapezoid $EFGH$ and its image $E'F'G'H'$.

**Example 5: Rotation**
A triangle with vertices $A (1, -3), B (3, 1)$, and $C (5, -2)$ is rotated $180^\circ$ about the origin.

a.) Find the coordinates of the vertices of the image.

b.) Graph triangle $ABC$ and its image $A'B'C'$. 