1.7: Logical Reasoning

HCPS III

- ***Standard 13:*** Data Analysis, Statistics, and Probability: DATA ANALYSIS: Develop and evaluate inferences, predictions, and arguments that are based on data

Objectives:

- Identify the ***hypothesis*** and ***conclusion*** in a conditional statement.
- Use a ***counterexample*** to show that an assertion is false.

Hypothesis and Conclusion

Conditional Statement: a statement written in the form **"if A, then B."**
(Also called "if-then" statements)

E.g., **If** the temperature is 100°C, **then** water will evaporate.

Hypothesis:
- the condition that needs to be met first
- it is part of a statement that follows the word "**IF**"

Conclusion:
- the result of the condition
- it is part of a statement that follows the word "**THEN**"

Example 1: Identify Hypothesis and Conclusion

Identify the hypothesis and conclusion of each statement.

a.) If it is raining, then Katy and Chloe will not play softball.

Hypothesis: **Conclusion:**

b.) If $7y + 5 \leq 26$, then $y \leq 3$.

Hypothesis: **Conclusion:**

Example 2: Write a Conditional in *if-then* Form

Identify the hypothesis and conclusion of each statement. Then write each statement in if-then form.

a.) I eat light meals.

Hypothesis: **Conclusion:**

If-Then Statement:

b.) For a number *x* such that $8 + 5x = 43, \ \ x = 7.$

Hypothesis: **Conclusion:**

If-Then Statement:

Counterexamples

Deductive Reasoning: the process of using facts, rules, definitions or properties to make a valid conclusion. If a conditional statement is true, then we can use it to make a valid conclusion.

Counterexample: a specific case in which a statement is false.

Example 3: Deductive Reasoning

Determine a valid conclusion that follows from the statement, "If one number is odd and another number is even, then their sum is odd" for the given conditions. If a valid conclusion does not follow, write *no valid conclusion* and explain why.

a.) The two numbers are 5 and 12.

b.) The two numbers are 8 and 26.

Example 4: Find a Counterexample

Provide a counterexample for each conditional statement.

a.) If Joe did not eat lunch, then he must not feel well.

b.) If the traffic light is red, then the cars must be stopped.

Example 5: Find a Counterexample

Which numbers are counterexamples for the statement below?

$x - y \neq y - x$

A. $x = 2, y = 3$
B. $x = 4, y = 4$
C. $x = 0, y = 1$
D. $x = 7, y = -7$